Calculation of Compressible Turbulent Boundary Layers on an Infinite Yawed Airfoil

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Theme

THE three-dimensional compressible turbulent boundary layer on an infinite yawed airfoil is a problem which has received little attention in the literature. Needless to say, the infinite yawed airfoil never occurs in practical applications (e.g., a transport aircraft). The importance of this type flow concerns the trends which it reveals (i.e., the effects of three-dimensional flow on turbulent boundary-layer characteristics). In addition, high aspect ratio finite-length swept airfoils may be adequately represented by an infinite yawed wing in regions removed from the wing root and tip.[1]

A theoretical attempt at modeling the three-dimensional compressible turbulent boundary layer on an infinite yawed airfoil based upon momentum integral and entrainment equation methods has recently been reported by Smith.[2] The current paper addresses the same problem through implicit finite-difference numerical integration of the governing turbulent boundary-layer equations in conjunction with a scalar eddy viscosity model of three-dimensional turbulence.

Contents

The present analysis employs the three-dimensional compressible turbulent boundary-layer equations in terms of time-averaged mean-flow quantities as derived by Vaglio-Laurin.[3] Assuming the ratio of boundary-layer thickness to local surface curvature to be everywhere small, the governing equations of motion, in terms of the orthogonal coordinate system x,y,z illustrated in Fig. 1, reduce to the following:

Continuity

$$\frac{\partial\bar\rho\bar u}{\partial x} + \frac{\partial\bar\rho V}{\partial y} + \frac{\partial\bar\rho\bar w}{\partial z} = 0 \tag{1}$$

x-Momentum

$$\bar\rho\bar u\frac{\partial\bar u}{\partial x} + \bar\rho V\frac{\partial\bar u}{\partial y} + \bar\rho\bar w\frac{\partial\bar u}{\partial z} = -\frac{\partial\bar p}{\partial x} + \frac{\partial}{\partial y}\left[\mu\frac{\partial\bar u}{\partial y} - \bar\rho\overline{u'v'}\right] \tag{2}$$

y-Momentum

$$\partial\bar p/\partial y = 0 \tag{3}$$

z-Momentum

$$\bar\rho\bar u\frac{\partial\bar w}{\partial x} + \bar\rho V\frac{\partial\bar w}{\partial y} + \bar\rho\bar w\frac{\partial\bar w}{\partial z} = -\frac{\partial\bar p}{\partial z} + \frac{\partial}{\partial y}\left[\mu\frac{\partial\bar w}{\partial y} - \bar\rho\,\overline{v'w'}\right] \tag{4}$$

Energy

$$\bar\rho\bar u\frac{\partial\bar H}{\partial x} + \bar\rho V\frac{\partial\bar H}{\partial y} + \bar\rho\bar w\frac{\partial\bar H}{\partial z} = \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial\bar H}{\partial y} + \frac{1-Pr}{Pr}\frac{\partial\bar h}{\partial y}\right) - \bar\rho\,\overline{v'H'}\right] \tag{5}$$

where

$$V = \bar v + (\overline{\rho'v'}/\bar\rho) \tag{6}$$

$$H = \bar h + (\bar u^2 + \bar w^2)/2 \tag{7}$$

Implicit in the preceding equations is the requirement of an infinite extent body of the yawed wing type which leads to the term $\partial/\partial z = 0$.

A thermally and calorically perfect air model is used in the present work. The laminar viscosity is taken to obey Sutherland's law while the laminar Prandtl number is assumed to have a constant value of 0.71. Three-dimensional turbulence is accounted for through the scalar eddy viscosity-mixing length model of Adams[4,5] in conjunction with an assumed constant turbulent Prandtl number (based on the use of static enthalpy) of 0.90 across the entire boundary layer.

The governing boundary-layer equations are transformed to Illingworth-Levy variables which allow linearized finite-

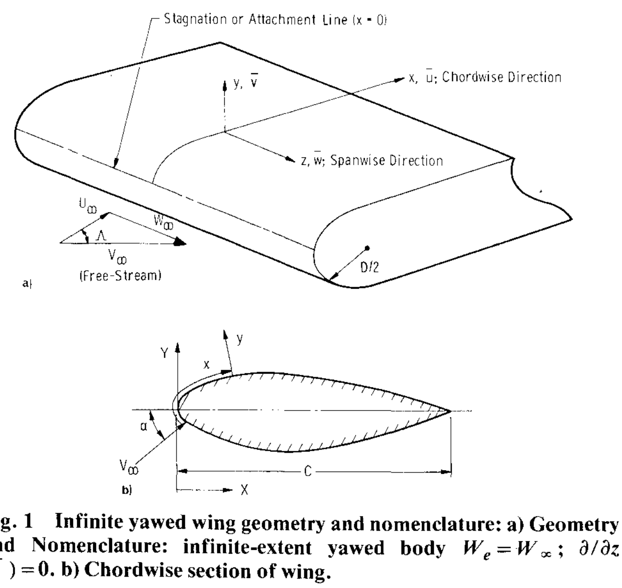

Fig. 1 Infinite yawed wing geometry and nomenclature: a) Geometry and Nomenclature: infinite-extent yawed body $W_e = W_\infty$; $\partial/\partial z\,(\bar{\ }) = 0$. b) Chordwise section of wing.

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Index categories: Subsonic and Transonic Flow; Boundary Layers and Convective Heat Transfer—Turbulent.

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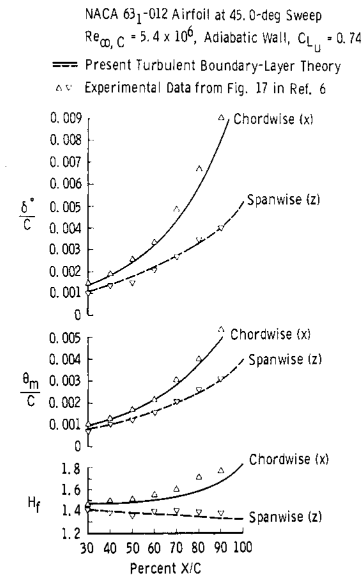

Fig. 2 Turbulent boundary-layer parameters on a lifting infinite yawed airfoil.

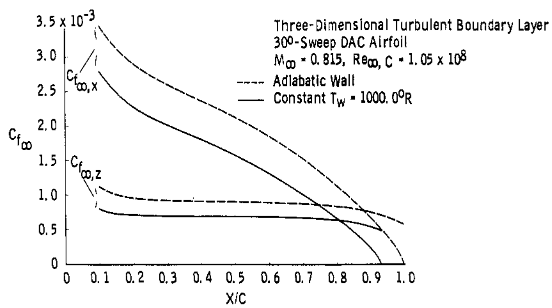

Fig. 3 Hot-wall effects on skin-friction coefficient distributions.

difference equivalents of the equations to be easily formulated. These are then solved using an iterative, marching, implicit finite-difference integration technique involving inversion of tridiagonal matrices. Available digital computer program options include calculation of the laminar or turbulent boundary layer on the stagnation or attachment line, instantaneous step-jump transition from laminar to turbulent flow at a prescribed location, variable grid node point spacing in both the chordwise and body normal directions, and prescribed arbitrary wall temperature distributions or an adiabatic wall. Classical sweep theory is used to provide the inviscid flow parameters needed as input for the boundary-layer outer edge conditions.

Probably the most complete and best-documented set of experimental data for subsonic flow over an infinite yawed airfoil is that reported by Altman and Hayter[6] some twenty years ago. This experimental investigation was conducted in the Ames Aeronautical Laboratory 7- by 10-ft subsonic wind tunnel using both swept and unswept NACA 63$_1$-012 section airfoils. The swept wing, having a sweep angle of 45° and a chord of 2.5 ft perpendicular to the leading edge, was mounted horizontally and spanned the 10-ft dimension of the wind tunnel. A comparison of calculated and experimental turbulent boundary-layer parameters (displacement thickness, δ^*, momentum thickness, θ_m, and shape factor, H_f) for both the chordwise (x-direction) and spanwise (z-direction) components of flow over the upper surface of the airfoil is given in Fig. 2. Boundary-layer transition from laminar to turbulent flow is taken to occur near the leading edge of the airfoil in accord with the experimental results of Ref. 6. As can be seen from Fig. 2, agreement between the present analysis technique and experiment is, in general, good over the entire airfoil with the largest discrepancy in the chordwise displacement thickness near the trailing edge.

Figure 3 presents the result of a hot-(1000°R) vs adiabatic-wall calculation for the case of high Reynolds numbers transonic flow over the upper surface of the Douglas Aircraft Company (DAC) 30-deg-sweep airfoil given in Fig. 15 of Ref. 7. The chordwise (x-direction) skin friction for the hot wall condition is reduced by some 20 to 40% over the corresponding location x-direction adiabatic wall value. Similarly, the spanwise (z-direction) hot-wall skin friction is reduced some 20 to 25% over the entire airfoil as compared with the corresponding location z-direction adiabatic-wall value. Needless to say, this significant decrease in local skin friction due to the heated wall can result in appreciable reduction of integrated skin-friction drag on the airfoil. This finding has application to the transonic portion of lifting vehicle re-entry where the wing surface temperature may reach soak values on the order of twice the freestream stagnation temperature because of the hypersonic high-heating phase of the re-entry trajectory.

References

[1]Hicks, J. G., and Nash, J. F., "The Calculation of Three-Dimensional Turbulent Boundary Layers on Helicopter Rotors," CR-1845, May 1971, NASA.

[2]Smith, P. D., "A Calculation Method for the Turbulent Boundary Layer on an Infinite Yawed Wing in Compressible, Adiabatic Flow," RAE TR 72193, Dec. 1972, Royal Aircraft Establishment, Farnborough, England.

[3]Vaglio-Laurin, R., "Turbulent Heat Transfer on Blunt-Nosed Bodies in Two-Dimensional and General Three-Dimensional Hypersonic Flow," *Journal of Aeronautical Sciences,* Vol. 27, Jan. 1960, pp. 27-36.

[4]Adams, J. C., Jr., "Finite-Difference Analysis of the Three-Dimensional Turbulent Boundary Layer on a Sharp Cone at Angle of Attack in a Supersonic Flow," AIAA Paper 72-186, San Diego, Calif., 1972.

[5]Adams, J. C., Jr., "Analysis of the Three-Dimensional Compressible Turbulent Boundary Layer on a Sharp Cone at Incidence in Supersonic and Hypersonic Flow," AEDC-TR-72-66, June 1972, Arnold Engineering Development Center, Arnold Air Force Station, Tenn.

[6]Altman, J. M. and Hayter, N. F., "A Comparison of the Turbulent Boundary-Layer Growth on an Unswept and a Swept Wing," TN 2500, Sept. 1951, NACA.

[7]Kaups, K. and Keltner, G., "Laminar Compressible Boundary Layer on a Yawed Infinite Wing," Rept. LB32706, March 1967, Douglas Aircraft Co., Long Beach, Calif.